COAXIAL HEAT PIPES

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Heat and mass exchange is calculated for the wicks of coaxial heat pipes during convective heat transfer between a fluid and the pipe wall.

For cooling and heat stabilization of cylindrical devices such as oscillator tubes, klystrons, and travelling-wave tubes, it is convenient to use coaxial heat pipes in which heat transfer is radial [1]. The design of a coaxial pipe for selection of the best combination of porous wick and coolant for given geometric pipe dimensions and thermal flux can be carried out by a method similar to that in [2, 3].

In a coaxial heat pipe, the heat source can be located either on the external surface or on the internal surface (Fig. 1). In particular, when the heat source is located on the internal surface (oscillator tube collector, liquid or gas flow), the heat flux can be transported without great loss in the radial direction to the external surface of the pipe and dissipated into the surrounding medium.

In such a pipe (Fig. 1), there are three sections of porous wick along which fluid transport occurs: 1) a wick in the region of vapor condensation; 2) a wick in the form of a disk connecting the evaporation and condensation regions radially, which can be considered an adiabatic region of the pipe; 3) a wick in the evaporation region.

In a coaxial heat pipe, the porous disks (adiabatic regions) are located at a spacing $2L_K = 2L_e$ and separate the pipe into a number of independent segments. If one considers heat and mass transfer in one of these segments, it is then sufficient to multiply the resultant value of the heat flux q by the area of the entire condenser or evaporator surface in order to determine the amount of heat transferred in the entire pipe.

To calculate the heat and mass transfer in the wick of a coaxial heat pipe, it is necessary to make the following assumptions.

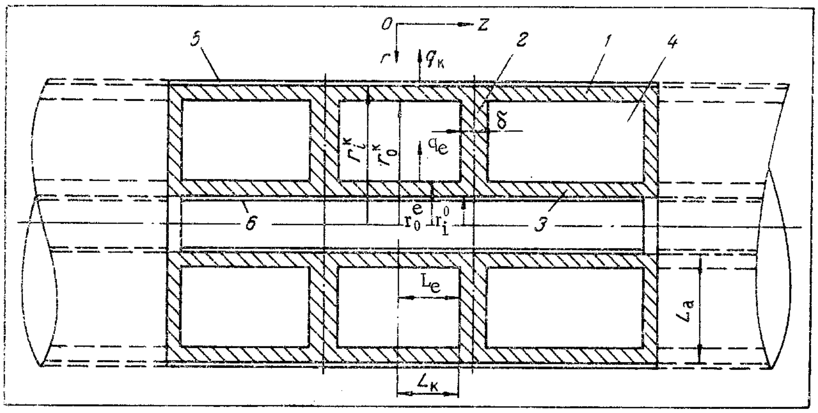

Fig. 1. Diagram of coaxial pipe: 1) wick of condenser element; 2) adiabatic zone; 3) wick of evaporator element; 4) vapor channel; 5) pipe wall in condenser zone; 6) pipe wall in evaporator zone.

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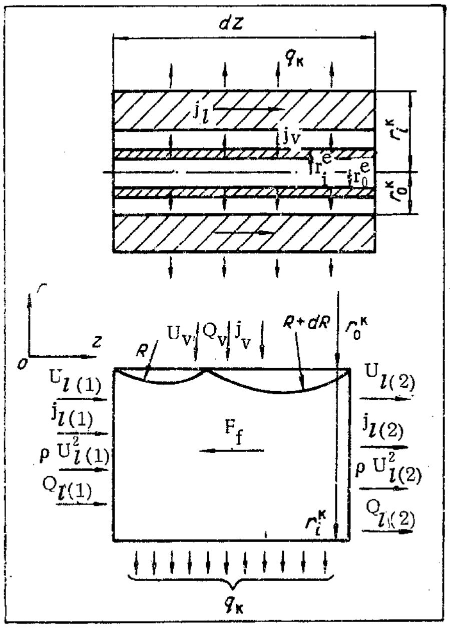

Fig. 2. Element of condenser section of coaxial heat pipe with length dz and thickness $r_i^K - r_0^K$.

1) The wick is incompressible and is of constant thickness in the sections L_K, L_a and L_e. The porous capillary body is isotropic, and in any cross section the pore area of the wick, S_p, and the total area S maintain the relation $S_p/S = \Pi$.

A similar relation is maintained for volume porosity.

2) The vapor and liquid over the entire condenser length L_k and evaporator length L_e are at constant temperature; there is no supercooling of superheating of the liquid and the vapor pressure P_v = const.

3) The vapor is condensed at the liquid-vapor boundary and has a velocity U_v in a direction normal to the surface, i.e., the velocity U_v has no component along the z axis; accordingly, there is no change in momentum along the axis.

4) The velocity of the liquid flowing in the porous wick, U_l, has only a z component, is constant over the entire thickness of the wick and is equal to the average velocity of the liquid in a single capillary, i.e., is determined by the Darcy's law.

5) We neglect the effect of the gravitational field.

Since the porous toroidal sleeves (adiabatic zones) separate the coaxial pipe into a number of independent sections, it is sufficient to consider half of such a section since the identical process occurs in the second half also. The length of a half-section in the condenser region is L_K, and in the evaporator region, L_e (Fig. 1).

6) We also neglect all terms containing differentials of second and higher orders because of their smallness.

7) There is no liquid film on the surface of the wick; vapor condensation takes place directly in the pores and evaporation is from the wick pores.

We shall consider in order the three elements of a half-section of the wick, beginning with the condenser. We assume the velocity of the liquid is zero at the point 0, the junction of the two half-sections.

1) Condenser. A section of the coaxial pipe is sketched in Fig. 1. We consider mass and energy balance for an element of the porous condenser of length dz, external diameter $2r_i^K$ and internal diameter $2r_0^K$ (Fig. 2).

a) Mass balance:

$$j_{l(1)} + j_v = j_{l(2)}; \tag{1}$$

$$j_{l(1)} = \rho_l \pi (r_i^{K^2} - r_0^{K^2}) U_l \Pi; \tag{2}$$

$$U_{l(2)} = U_{l(1)} + dU_l; \tag{3}$$

$$j_{l(2)} = \rho_l \Pi\pi (r_i^{K^2} - r_0^{K^2}) U_{l2} = \rho_l \Pi\pi (r_i^{K^2} - r_0^{K^2})(U_{l_1} \quad dU_{l_1}); \tag{4}$$

$$j_v = \rho_v U_v \Pi 2\pi r_0^K dz = j_{l(2)} - j_{l(1)} = \rho_l \Pi\pi (r_i^{K^2} - r_0^{K^2}) \frac{dU_l}{dz} dz; \tag{5}$$

$$F_{p1} - F_{p2} - F_f = \rho_l U_1^2 \Pi\pi (r_i^{K^2} - r_0^{K^2}) - \rho_l U_2^2 \Pi\pi (r_i^{K^2} - r_0^{K^2})$$

$$= \rho \frac{d(U_l^2)}{dz} dz \Pi\pi (r_i^{K^2} - r_0^{K^2}). \tag{6}$$

In accordance with Darcy's law:

$$F_f = K_1 \frac{\mu_l}{\rho_l} j_l \Pi dz = K_1 \Pi^2 \pi (r_i^{K^2} - r_0^{K^2}) \mu_l U_l dz; \tag{7}$$

$$F_{p1} = \left(P_v - \frac{2\sigma}{R}\right) \Pi\pi (r_i^{K^2} - r_0^{K^2}); \tag{8}$$

$$F_{p2} = \left(P_v - \frac{2\sigma}{R + dR}\right) \Pi\pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right). \quad (9)$$

Substituting Eqs. (7)-(9) into Eq. (6), we obtain

$$-2\sigma \frac{dR}{R^2} - \Pi K_1 \mu_l U_l dz = \rho_l \frac{d(U_l^2)}{dz} dz. \quad (10)$$

b) Energy Balance. We shall consider energy balance in an element of the porous condenser of a coaxial heat pipe assuming that the thermal energy in the porous wick is transferred by convection and therefore neglect heat transfer by thermal conduction:

$$Q_{l(1)} + Q_v = Q_{l(2)} + Q_1; \quad (11)$$

$$Q_{l(1)} = j_l h_l = h_l \rho_l \Pi\pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right) U_{l1}; \quad (12)$$

$$Q_v = j_v h_v = h_v \rho_l \Pi\pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right) \frac{dU_l}{dz} dz; \quad (13)$$

$$Q_{l(2)} = j_l h_l + \frac{d(j_l h_l)}{dz} dz = h_l \rho_l \Pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right) \pi \left(U_l + \frac{dU_l}{dz} dz\right); \quad (14)$$

$$Q_1 = q 2\pi r_i^{\kappa} dz. \quad (15)$$

Summing Eqs. (12)-(14) and (15), we obtain

$$(h_v - h_l) \rho_l \Pi\pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right) \frac{dU_l}{dz} = \rho_l \Pi\pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right) U_l \cdot \frac{dh_l}{dz} + q 2\pi r_i^{\kappa}; \quad (16)$$

$$U_l \frac{dh_l}{dz} \rho_l \Pi\pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right) - (h_v - h_l) \rho_l \Pi\pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right) \frac{dh_l}{dz} + q 2\pi r_i^{\kappa} = 0. \quad (17)$$

Since $(dh_l/dz) = 0$ because of assumption (2), we have:

$$r' \rho_l \Pi\pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right) \frac{dU_l}{dz} = q 2\pi r_i^{\kappa}; \quad (18)$$

$$\frac{dU_l}{dz} = \frac{2q\pi r_i^{\kappa}}{r' \rho_l \Pi\pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right)}. \quad (19)$$

Integrating Eq. (19) from 0 to L_K, we obtain

$$U_l = \frac{2q r_i^{\kappa}}{r' \rho_l \Pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right)} L_{\kappa}, \quad (20)$$

$$j_l = \rho_l \Pi\pi \left(r_i^{\kappa^2} - r_0^{\kappa^2}\right) U_l = \frac{2q\pi r_i^{\kappa}}{r'} L_{\kappa}. \quad (21)$$

2) Adiabatic Region. The adiabatic region is in the form of a disk of radius r_0^K and thickness δ having a central opening of radius r_0^e adjoined by the porous evaporator. The adiabatic region has a coefficient of resistance $K_2 \ll K_1$.

Transfer of liquid through the adiabatic region in the radial direction from condenser to evaporator follows the Darcy's law

$$j_l = -\frac{2\pi\delta \left(\frac{2\sigma}{R_i^{\kappa}} - \frac{2\sigma}{R_i^{e}}\right) \rho_l}{K_2 \ln \left(r_0^{\kappa} / r_0^{e}\right) \mu_l}. \quad (22)$$

The pressure drop because of the difference in curvature of the liquid-vapor boundary at the end of the condenser and in the adiabatic region, which causes the liquid to move, can be determined by equating the flow (21) out of the condensation region to the flow (22) which passes through the adiabatic region:

$$\frac{2\sigma}{R_i^{\kappa}} - \frac{2\sigma}{R_i^{e}} = \frac{K_2 q_{\kappa} r_i^{\kappa} L_{\kappa} \ln \left(r_0^{\kappa} / r_0^{e}\right) \mu_l}{\rho_l r' \delta} \quad (23)$$

3) Evaporator. The evaporator of a coaxial heat pipe has the form of a hollow cylinder with the geometric dimensions r_i^e, r_0^e, and L_e.

In the plane $z = L_e$, the rate of liquid transport through the porous wick is

$$U_{l\,(z=L_e)} = \frac{jl}{\rho_l \Pi \pi (r_i^{2e} - r_0^{2e})} = \frac{2q_\text{к} r_i^\text{к} L_\text{к}}{\Pi (r_i^{2e} - r_0^{2e}) r' \rho_l} . \tag{24}$$

If it is assumed the thermal flux is supplied uniformly to the entire area of the evaporator and evaporation occurs from the surface of the porous wick, then, as in the condenser, the velocity of the liquid in the pores of the wick in the z direction will be proportional to z. Thus

$$\frac{U_{l\,(z=L_e)}}{U_{l\,(z)}} = \frac{L_e}{L_e - z} ; \tag{25}$$

$$U_{l\,(z)} = \frac{2q_\text{к} r_i^\text{к} L_\text{к} (L_e - z)}{\Pi (r_i^{2e} - r_0^{2e}) r' \rho_l L_e} . \tag{26}$$

The general integral equations for conservation of energy, mass, and momentum in the condenser and evaporator of the heat pipe have the form

$$-\int\limits_{R_{z=0}}^{R_{z=L_\text{к}}} 2\sigma \frac{dR}{R^2} - \int\limits_0^{L_\text{к}} K_1 \mu_l \frac{2q_\text{к} r_i^\text{к}}{r' \rho_l (r_i^{2\text{к}} - r_0^{2\text{к}})} z dz = \int\limits_0^{L_\text{к}} \frac{4q_\text{к}^2 r_i^{2\text{к}}}{r'^2 \rho_l \Pi^2 (r_i^{2\text{к}} - r_0^{2\text{к}})^2} z dz; \tag{27}$$

$$-\int\limits_{R_{z=L_e}}^{R_{z=0}} 2\sigma \frac{dR}{R^2} - \int\limits_{L_e}^{0} K_1 \mu_l \frac{2q_\text{к} r_i^\text{к} L_\text{к}}{r' \rho_l (r_i^{2e} - r_0^{2e})} \frac{(L_e - z)}{L_e} dz = \int\limits_{L_e}^{0} \frac{4q_\text{к}^2 r_i^{2\text{к}} L_\text{к}^2}{\Pi^2 (r_i^{2\text{и}} - r_0^{2\text{и}}) r'^2 \rho_l L_e^2} \frac{d(L_e - z)^2}{dz} dz. \tag{28}$$

If we integrate Eqs. (27) and (28) and add them, we obtain, by using Eq. (23),

$$\frac{2\sigma}{R_{min}} + q_\text{к} \frac{\mu_l r_i^\text{к} L_\text{к}}{\rho_l r'} \left[\frac{K_2 \ln (r_0^\text{к}/r_0^e)}{\delta} - \frac{K_1 L_e}{(r_i^{2e} - r_0^{2e})} + \frac{K_1 L_\text{к}}{(r_i^{2\text{к}} - r_0^{2\text{к}})} \right] = \frac{2q_\text{к}^2 r_i^{2\text{к}} L_\text{к}^2}{\Pi^2 r'^2 \rho_l} \left[\frac{1}{(r_i^{2e} - r_0^{2e})^2} - \frac{1}{(r_i^{2\text{к}} - r_0^{2\text{к}})^2} \right]. \tag{29}$$

We introduce the following notation:

$$A = \left[\frac{K_2 \ln (r_0^\text{к}/r_0^e)}{\delta} - K_1 \left(\frac{L_e}{r_i^{2e} - r_0^{2e}} - \frac{L_\text{к}}{r_i^{2\text{к}} - r_0^{2\text{к}}} \right) \right];$$

$$B = \frac{1}{(r_i^{2e} - r_0^{2e})^2} - \frac{1}{(r_i^{2\text{к}} - r_0^{2\text{к}})^2};$$

$$C = \left[\frac{K_1 r' \mu_l \Pi^2}{2q_\text{к} r_i^\text{к}} \left(\frac{1}{r_i^{2\text{к}} - r_0^{2\text{к}}} \right) - \frac{1}{(r_i^{2e} - r_0^{2e})^2} - \frac{1}{(r_i^{2\text{к}} - r_0^{2\text{к}})^2} \right];$$

$$D = \left[K_2 \frac{\ln (r_0^\text{к}/r_0^e)}{\delta} + K_1 \frac{L_e}{r_i^{2e} - r_0^{2e}} \right].$$

Equation (29) can be solved with respect to L_K or q_K:

$$L_\text{к} = -\frac{\mu_l r' \Pi^2 D}{4 r_i^\text{к} q_\text{к} C} + \sqrt{\left(\frac{r' \mu_l \Pi^2 D}{4 r_i^\text{к} q_\text{к} C} \right)^2 + \frac{\sigma r'^2 \rho_l \Pi^2}{r_i^{2\text{к}} q_\text{к}^2 C R_{min}}} ; \tag{30}$$

$$q_\text{к} = -\frac{\mu_l r' \Pi^2 A}{4 r_i^\text{к} L_\text{к} B} + \sqrt{\left(\frac{r' \mu_l \Pi^2 A}{4 r_i^\text{к} L_\text{к} B} \right)^2 + \frac{\sigma r'^2 \rho_l \Pi^2}{r_i^{2\text{к}} L_\text{к}^2 B R_{min}}} . \tag{31}$$

Since the entire heat pipe consists of n sections, we have

$$L_p = 2 L_\text{к} n. \tag{32}$$

The thermal flux in the evaporator is

$$q_e = -\frac{q_K r_i^K L_K}{r_0^e L_e} . \tag{33}$$

The thermal power fed to the evaporator and dissipated by the condenser is

$$Q = -q_e 2\pi r_0^e L_p = -q_K 2\pi r_i^K L_p .$$

The method of calculating the section length L_K for a cylindrical condenser in a coaxial heat pipe with a given q_K, the type of coolant, the transport characteristics of the wick and its geometric dimensions can be illustrated by the following example.

Let the porous wick of the heat pipe in the evaporation and condensation regions have the following dimensions: $r_i^K = 4 \cdot 10^{-2}$ m, $r_0^K = 3.9 \cdot 10^{-2}$ m, $r_i^e = 1.1 \cdot 10^{-2}$ m, $r_0^e = 10^{-2}$ m. Its permeability is $K = 1/K_1 = 0.16 \cdot 10^{-9}$ m^2; $R_{min} = 3 \cdot 10^{-4}$ m, and the porosity $\Pi = 0.7$.

We assume the permeability of the adiabatic region is considerably greater than the permeability of the wick in the evaporation and condensation regions, $K_2 \ll K_1$, so that it need not be considered in the calculations.

The coolant characteristics are: $\rho_l = 0.79 \cdot 10^3$ kg/m^3; $\sigma = 18.3 \cdot 10^3$ J/m^2; $\mu_l = 1.2 \cdot 10^{-3}$ kg/m-sec; $r' = 1.1 \cdot 10^6$ J/kg.

Let the given thermal flux be $q_K = 10^4$ W/m^2:

$$A = 6.2 \cdot 10^4 \text{ m}^{-2}; \qquad B = 26.6 \cdot 10^8 \text{ m}^2.$$

For the conditions given, the length of a condenser section is

$$L_K = 0.14 \text{ m}.$$

The total thermal power dissipated by a condenser section is

$$Q_K = 680 \text{ W}.$$

NOTATION

σ	is the surface tension coefficient;
R_{min}	is the minimum radius of curvature of liquid-vapor interface;
K_2	is the hydraulic resistance of the adiabatic zone;
K	is the permeability of the porous wick;
$K_1 = 1/K$	is the hydraulic resistance of the porous wick;
ρ_l	is the liquid density;
r'	is the latent heat of evaporation;
μ_l	is the liquid viscosity;
Π	is the porosity;
L_K	is the length of the condenser element;
L_e	is the length of the evaporator element;
δ	is the thickness of the adiabatic zone;
j_l, j_v	are the liquid and vapor flows;
U_v, U_l	are the vapor and liquid velocities;
F_f	is the friction force;
F_{p_1}, F_{p_2}	are the pressure forces at points 1 and 2;
Q	is the heat power;
h_v, h_l	are the vapor and liquid enthalpies;
q	is the heat flux;
R	is the pore radius;
Q_{eT}, Q_{KT}	the total heat power transferred in evaporator and condenser of heat pipe;
L_p	is the length of the heat pipe.

LITERATURE CITED

1. L. L. Vasil'ev, Report at the IV All-Union Conference on Heat and Mass Transfer [in Russian], Vol. 2, Part 2, Minsk (1972).
2. H. R. Kunz, L. S. Langston, B. H. Hilton, S. S. Wyde, and G. H. Nashick, Vapor-Chamber Fin Studies, NASA CR-812.
3. L. L. Vasil'ev and S. V. Konev, Inzh.-Fiz. Zh., 20, No. 3 (1971).